

On the influence of Mountain-Attraction on the determination of the relative heights of Mount Everest, near Darjeeling, and the lofty peak lately discovered near Kashmir.

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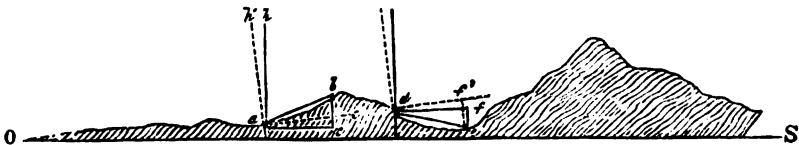
In the communication read at the July meeting of the Asiatic Society by Major Thuillier, the interesting fact was stated that a mountain has been found in the neighbourhood of Kashmir (about 36° North latitude and $76\frac{1}{2}^{\circ}$ East longitude), of which the height does not fall far short of Mount Everest (Lat. 28° , Long. 87°), the highest known mountain in the world, and which towers up to 29,002 feet or $5\frac{1}{2}$ miles above the level of the sea. The newly discovered peak is only 724 feet lower than this, and is 122 feet higher than Kunchinjunga, the highest known before the discovery of Mount Everest. And hopes are held out that before the survey of the hills in the neighbourhood is completed, some other mountain may yet be found in that western extremity of the range to rear its head as high even as the monarch of the east.

2. In the coming contest, then, for the sovereignty between the East and West of this stupendous range of mountains even a *small* circumstance may give the palm to one or the other. It is with this feeling that I lay the following statement before the Society.

In itself the precise determination of the height of a mountain is a matter of little importance. It is not to be compared, in a scientific point of view, with the importance of obtaining correct *horizontal* measures and the correct curvature of the arcs measured. But where mountains are contending for the pre-eminence of being the highest in the whole world, the question assumes special interest.

3. I take it for granted, that, as the effect of Mountain-Attraction on the levelling of the instruments of observation has not been taken account of in the Survey of the Plains, the same course has been followed in the Survey of the Mountains. It is to the effect which this disturbing cause must have upon the measurement of the heights in question, that I wish to call attention. My results will be only approximations: but I believe they are sufficient to show the tendency of things, and to add one more illustration to

others, I have elsewhere given, of the importance of means being taken to calculate the effects of this disturbing cause more completely.



4. The diagram above is an ideal vertical section of the plains and mountains, intended merely to illustrate the effect of Mountain-Attraction upon the determination of the heights.

OS is the sea-level, (lying on that spheroidal surface, of ellipticity $\frac{1}{300}$, of which the Ocean is supposed to form a part). To this level all heights are referred in the Survey: a and b are two stations of observation, ah the vertical at a perpendicular to the sea-level: the height of b above a is determined by the Survey, and this being done at each succeeding station the height of the highest peak is found by adding together the successive changes in height. In this diagram I have supposed all the stations of observation, leading from the sea up to the highest peak, to lie in the same vertical plane. This is not the case, some will lie on one side and some on the other. But taking this into account would make no difference in my results.

Draw ac parallel to the sea-level and bc perpendicular to it. Then bc is the true height of b above a . But the plumb-line will not hang in the line ha , but in another line $h'a$, owing to the Attraction of the Mountains: and therefore the spirit-level will make ac' (at right angles to ah') the apparent level line at a , and not ac . Hence, if bc' be at right angles to ac' , bc' is the height of b above a , as brought out by the Survey. This is too small by $bc - bc'$.

Let the angle $h'ah = v$, and angle $bac = \theta$. Then

$$\begin{aligned} bc - bc' &= ab \sin \theta - ab \sin (\theta - v) \\ &= ab \sin \theta - ab \sin \theta \cos v + ab \cos \theta \sin v \\ &= ab \cos \theta \text{ arc } v'', \text{ because } v \text{ is very small} \\ &= ac \times \text{arc } v'' \end{aligned}$$

Hence Mountain-attraction, if not corrected in the calculation, will have the effect of making b less high above a than it really is, by a space $= ac \times \text{arc } v''$.

5. The same will be the case where the next station (as e) is lower, instead of higher, than the station from which the observation is made (as d). The Survey makes the distance of e below $d = ef'$, whereas it really is ef . Hence the effect of Mountain-Attraction is, as before, to make the successive heights above the sea-level too small.

6. I will now endeavour to approximate to the aggregate effect of this disturbing cause upon the heights of Mount Everest in the East and of the newly discovered Mountain in the West.

In communications to the Royal Society (see *Phil. Trans.* 1855 and 1859) I have shown that if through a point in the meridian of Cape Comorin and in latitude 33° a straight line be drawn in a direction E. S. E., that line may be regarded as an Axis of the Himalayas; such that the Mountain-Mass attracts places in the plains with a force varying inversely as the distance from that axis, at any rate for stations lying between the foot of the hills and a distance of about 1000 miles from the axis.

Also it is shown that at a distance of 222 miles from this axis the deflection of the plumb-line towards the north is $28''$, and therefore in a direction at right angles to the axis $= 28'' \text{ sec. } 22^\circ 30' = 30''$. A line about parallel with the axis at a distance of 156 miles marks the average commencement of the plains. From this line, then, the law of the inverse distance according to which the deflections vary, may be supposed to begin. Within this limit, that is, within the hill-region, the law will be different. At the line itself, that is, at the foot of the hills, the deflection northwards will be $=$

222	222
— 28'' = 40'', and the deflection towards the axis =	— 30'' = 43''.
156	156

7. Assuming that these data hold good for the foot of the hills below Darjeeling and for those below Kashmir, I proceed to find the accumulated effect of the errors in height at a series of stations, connecting the nearest point of the sea, viz. the Sandheads due South, with the foot of the Darjeeling Hills, running over a space of about 360 miles: and then the same at a series of stations, con-

necting the nearest point of the sea, viz. the mouths of the Indus, with the foot of the Kashmir Hills, running over a space of about 720 miles.

8. In the case, then, of Mount Everest in the Darjeeling Hills, the line of stations runs up due north over 360 miles to the foot of the hills, from which the distance due north to the axis = $156 \times \sec. 22^\circ 30' = 168$ miles. Suppose the stations along this line are at 12 miles distance from each other in succession (which is about the average used in the Survey). There will be 30 such stations, at the distances 180, 192, 516 miles from the axis of the Himalayas, measured due north. Hence at these places the deflections of the plumb-line (which vary inversely as the distance from the axis) are

$$\frac{168}{180} 40'', \frac{168}{192} 40'', \dots \dots \dots \frac{168}{516} 40''.$$

Now by para. 4 the difference of level caused by a deflection $40'' = 12 \text{ miles} \times \text{arc } 40''$.

$$= 12 \times 3 \times 1760 \times \frac{40}{180 \times 60 \times 60} \pi = 12.3 \text{ feet.}$$

Hence, the aggregate change of level between the foot of the Darjeeling Hills, arising from this cause,

$$= 12.3 \left\{ \frac{14}{15} + \frac{14}{16} + \dots \dots \dots + \frac{14}{43} \right\} \text{ feet.}$$

$$= 172.2 \left\{ \frac{1}{15} + \frac{1}{16} + \dots \dots \dots + \frac{1}{43} \right\}$$

$$= 172.2 \times 1.098 = 189 \text{ feet.}$$

9. I will now find the change of level at the other extremity of the range. The range there slightly inclines more to the north. The axis of the Himalayas may therefore be taken, for those parts, to be a line drawn, as before, though a point in latitude 33° of the meridian of Cape Comorin, but inclined 30° north of west. This axis runs near Skardo; and is, as before, about 156 miles from the foot of the hills; which are about 720 miles from the mouths

of the Indus, the nearest point of the sea. The line joining the sea with the foot of the hills passes over 720 miles (or 60 stations at 12 miles apart) and is about perpendicular to the axis above described, which is 156 miles beyond the foot of the hills.

Pursuing the same course as before, I find that the aggregate change of level of the foot of the Kashmir Hills above the sea level, owing to Mountain-Attraction not being taken into consideration,

$$\begin{aligned}
 &= 12.3 \times \frac{43''}{40''} \left\{ \frac{156}{168} + \frac{156}{180} + \dots + \frac{156}{876} \right\} \\
 &= 12.3 \times \frac{43}{40} \times 13 \left\{ \frac{1}{14} + \frac{1}{15} + \dots + \frac{1}{73} \right\}^* \\
 &= 171.9 \times 1.694 = 291.2 \text{ feet.}
 \end{aligned}$$

These calculations, then, if correct, show that the plains at the foot of the Darjeeling Hills are higher above the sea-level than the Survey makes them by 189 feet: and that the plains at the foot of the Kashmir Hills are higher above the sea-level than the Survey makes them by 291 feet. This gives 102 feet in favour of the plains near Kashmir above those near Darjeeling, arising from this cause.

10. I have yet to take into account the effect of Mountain-Attraction on the change of level along the line of stations connecting the plains with the mountains of which the altitude is under consideration. The law of deflection begins to alter from that of the inverse distance as soon as we enter the mass itself—just, as is well known, in the case of a sphere; a point outside it is attracted with a force varying inversely as the square of the distance from its centre, and therefore increasing as the point approaches: but as soon as the point enters the sphere it is attracted with a force varying directly as the distance from the centre, and therefore diminishing: the attraction at the surface of the sphere is greater than on any point outside or inside.

* I may observe that I have summed this and the previous harmonic series by using a table of cosines and secants, and adding up the cosines of the angles of which the secants are 14, 15, 16, &c.

So with the Himmalyan Mass. Its attraction on points outside the mass is shown (within certain limits) to be inversely as the distance from a given fixed line. But when we take a station within the mountain region this law must cease, and some other one come into operation. I have not the means of ascertaining what that law is. But, whatever it may be, it seems probable that it will be much the same as we pass in among the Kashmir Mountains, as it is in passing in among the Darjeeling Mountains. But the distance of the newly-discovered Mountain near Kashmir is about 270 miles from the foot of the hills, whereas Mount Everest is only about 100 miles. This circumstance must of itself give a great advantage to the Kashmir heights over those of the east end of the range.

11. In the above calculations I have considered the effect of the attraction of the Mountain Mass lying on the north of India. It is possible, that other causes may exist which either increase or moderate this effect. When any such cause is found its influence should be ascertained. One cause, besides the attraction of the north, lies in the deficiency of matter in the vast ocean lying on the south of India. This operates in two ways: (1) by affecting the plumb-line and producing effects similar to those I have been considering: and (2) by changing the sea-level at the Sandheads and also at the mouths of the Indus, that is, at the commencement of each of the two series of stations I have supposed to connect the sea with the Mountains in question. In this latter effect also the mountains give their aid.

The first of these causes will, as in the case of Mountain-attraction, make the height of Kashmir greater than the Survey makes it relatively to the east end of the range; while the effect of the second is doubtful. I have shown in a Paper read in December last before the Royal Society (see Proceedings, No. 34, p. 599), that the sea-level at the mouths of the Indus is very probably about 500 feet higher than at Cape Comorin owing to this cause. But how much higher it is at the Sandheads than at Cape Comorin, I did not in that calculation determine, as it was not required for the purposes of the paper. I should imagine that it would be very much the same as at the mouths of the Indus, as the Sandheads

are situated with reference to neighbouring seas and the vast ocean very similarly to them.

12. In conclusion, my own persuasion is, that when sufficient data are obtained to make the calculation complete, it will be found that Mountain-attraction, combined with deficiency of attraction of the Ocean, so far affects the levelling of the instruments of observation, as to cause the Survey to bring out the height of the newly-discovered Mountain near Kashmir too low by 150 or 200 feet relatively to Mount Everest. And consequently, that if a peak be found in those western Mountains, of which the height is about 200 feet less than that of Mount Everest according to the Survey measures, I believe myself, that there will be good ground at least for hesitating before we pronounce finally which of them is really the king of the whole range.

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Calcutta, July 21st, 1859.